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Constitutive equations for inhomogeneous plastic flow and application to Lüders band propagation

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When deformation is not macroscopically homogeneous, the structure variables that define the mechanical state of a material depend on position and their space derivatives appear in the constitutive equations for plastic deformation. The general form of these equations in uniaxial deformation is written for the case of a single structure variable. Constitutive equations for deformation by Lüders bands are derived in which the density of mobile dislocations plays the role of the structure variable. The equations exemplify the general form of a constitutive law for inhomogeneous deformation. Strain and strain rate profiles in a tensile specimen traversed by a "steady state" Lüders band are obtained by integration of the constitutive equations and the band parameters are determined. Comparison with experimental results is quite favourable in spite of the simplifications introduced.

1. Introduction

Considerable attention has been given, in recent years, to the development of accurate constitutive equations of plasticity (e.g. [1, 2]). The general form of the equations for homogeneous plastic deformation is now well established [3–5]. The equations are first written for uniaxial deformation (tension and compression), since most of the experimental data is obtained from these types of tests, and then additional relations involving the stress and strain rate tensors are introduced in order to deal with the more general case of triaxial stress states [6–8], eventually including anisotropy [8, 9].

Specific forms of constitutive equations that have been proposed for individual materials or classes of materials under various uniaxial deformation regimes (e.g. [7, 10–14) usually contain only one structure viarable. In such cases, the constitutive law includes two equations. The first, is the rate equation

$$\dot{\epsilon} = f(\sigma, S, T) \tag{1}$$

where $\dot{\epsilon} = d\epsilon/dt$ is the plastic strain rate, σ is the applied stress, T is the temperature and S is the structure variable. In principle, higher order time derivatives of ϵ and σ can be introduced, but to simplify we shall admit Equation 1 for the rate equation*. The second equation gives the time rate of change of the structure variable, S:

$$\frac{dS}{dt} = g(\sigma, S, T). \tag{2}$$

In these equations, f and g are definite functions for each material. When written for a class of materials, f and g may depend on various parameters which can be regarded as structure independent properties of each material in the class. The time t is not among the variables on which f and g depend explicitly [15]. When the deformation is homogeneous, as we are assuming presently, the space coordinates do not appear because the inter-

*It is unlikely that $\dot{\epsilon}$ depends on $\dot{\sigma}$, at least for thermally activated deformation [14]. But inertial effects can justify the inclusion of $\ddot{\epsilon}$ in f.

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vening variables are independent of position. The strain ϵ can be obtained by integration of Equations 1 and 2 for any deformation path defined, for example, by $\sigma(t)$ or $\dot{\epsilon}(t)$. The strain is a path variable [10] and for this reason cannot appear in the general form of the constitutive law.

Strictly, deformation is not homogeneous at the atomic level, although it can be treated as homogeneous on a macroscopic scale. Frequently, however, deformation is not macroscopically homogeneous. Examples include most forming operations and tensile deformation after necking (the stress state varies from point to point) and the tensile deformation due to propagation of Lüders bands (even if the stress is assumed as uniform). In such cases, the space variables have to be introduced in the constitutive equations.

The purpose of this paper is twofold. First, we discuss the general form of the constitutive law applicable to nonhomogeneous deformation. This will be done for the case where only one structure parameter, S, and one Cartesian coordinate, x, have to be considered. The simplest example is a tensile test of a tappered specimen, provided we neglect the triaxiality of the stress and assume $\sigma = \sigma(x)$. If the specimen is initially homogeneous, S is initially independent of position, but this situation will be destroyed by deformation (Equation 2) and $S = S(x)$. In theoretical treatments of plastic instability and neck evolution in tension, this type of approach is frequently used (e.g. [16, 17]) but, as will be shown, the straightforward generalization of Equations 1 and 2, with $\sigma(x)$ and $S(x)$, may not be sufficient to describe nonhomogeneous deformation.

The other example of simple inhomogeneous deformation, to which special attention will be given in this paper, is the propagation of a Lüders band in a tensile specimen. A particular form of a constitutive law for Lüders band deformation will be derived, based on a simple dislocation model. In spite of the simplicity of the approach, the predictions that can be derived are in very good agreement with experimental observations.

2. The constitutive law for inhomogeneous deformation

What is the form of the constitutive law, that corresponds to a generalization of Equations 1 and 2, when one space coordinate (e.g. a Cartesian coordinate) has to be introduced to account for the inhomogeneity of deformation? It will be assumed that the temperature is uniform. The space coordinate, x, does not have to appear explicitly as a variable in the functions f and g in Equations 1 and 2, as noted by McCartney [15] in a discussion on the role of time and space coordinates in constitutive equations. But the fact that now $\sigma(x)$ and $S(x)$ implies that both $\dot{\epsilon}$ and the time rate of change of S, which we write as $\partial S/\partial t$, are functions of x. The question is to decide whether or not additional variables have to be introduced in the arguments of the functions f and g. Possible candidates are $\partial\sigma/\partial x$, $\partial S/\partial x$ and higher order space derivatives (mixed time and space derivatives of σ and S will be excluded, for the same reason as $\dot{\sigma}$ was excluded in Equations 1 and 2). The inclusion of $\partial\sigma/\partial x$, $\partial^2\sigma/\partial x^2$, ... could be justified to account for a stress concentration effect in a tensile specimen with non-uniform cross-section, but this situation will not be considered. We shall then introduce, as an additional variable, the derivative $\partial S/\partial x$ (and discard $\partial^2 S/\partial x^2$, ... for simplicity). The dependence of $\dot{\epsilon}$ on $\partial S/\partial x$ implies that the strain rate at x at the instant t depends not only on the local state (i.e. the value of S at x, t) but also on the state at neighbouring regions at the same instant. This does not necessarily mean that "information" from neighbouring regions is received instantaneously at x. An example will be given in Section 3.2 of a situation in which $\dot{\epsilon}$ depends in fact on $\partial S/\partial x$.

From the previous discussion we may conclude that the form of the constitutive equations for nonhomogeneous deformation in the simplest situations (one structure parameter, one space coordinate, uniform temperature) is the following:

$$\dot{\epsilon} = f\left(\sigma, S, \frac{\partial S}{\partial x}, T\right) \tag{3a}$$

$$\frac{\partial S}{\partial t} = g\left(\sigma, S, \frac{\partial S}{\partial x}, T\right) \tag{3b}$$

with $\sigma(x); S(x); T$

If $T(x)$, an equation describing the transport of energy (heat) has to be added to the constitutive equations, from which the variation of temperature with time and space can be derived. But it is unlikely that $\partial T/\partial x$ should be included in the arguments of f and g in Equations 3.

Auxiliary equations may be written for the total rate of elongation, $\dot{L} = \mathrm{d}L/\mathrm{d}t$ (L is the specimen length along x)

$$\dot{L} = \int_0^L \dot{\epsilon}(x)\,\mathrm{d}x \tag{4}$$

and for the stress at any point produced by an applied load P in the x direction:

$$\sigma(x) = \frac{P}{A(x)} \tag{5}$$

where $A(x)$ is the area of the cross section at x. Finally, the constancy of volume gives

$$\dot{\epsilon}(x) = -\frac{\dot{A}(x)}{A(x)}. \tag{6}$$

Equations 3 to 6 can be integrated to give $A(x)$, $\epsilon(x)$ and $S(x)$ as a function of time for a given deformation path $\dot{L}(t)$ or $P(t)$.

3. Lüders band propagation

In this section we introduce a simplified model for Lüders band propagation in a tensile specimen, which illustrates the general form (Equation 3) of the constitutive law for nonhomogeneous deformation. The model leads to very simple constitutive equations, which can be integrated to obtain the strain rate profile in a steady state Lüders band and the various band parameters. The predictions of the model will be compared with experimental results of the literature.

3.1. The model and the constitutive law

It is assumed that the band propagates under a constant applied load and the tensile stress σ is independent of position x in the specimen. That is, both the area change due to the passage of the band and the consequent triaxility of the stress in the specimen will be neglected.

For the rate equation we use the simple form

$$\dot{\epsilon} = \rho b v \tag{7}$$

which relates the strain rate at x to the local density ρ of mobile dislocations of Burgers vector **b** and average velocity v. This velocity is assumed as a constant (under constant σ); ρ is the structure variable. To take into account the variation of v with x, which may well occur in Lüders band propagation even at constant σ, it would be necessary to introduce the total dislocation density and write two additional relations, respectively, between this density and ρ and between the total density and the accumulated strain, as in the treatment of Sändstrom and Lagneborg [18] of discontinuous yielding. This would complicate considerably the constitutive equations and would require some guesses about the form of those relations. A geometrical constant factor has been neglected in the second term of Equation 7.

With these simplifications, we now turn to the variation of the structure variable ρ with time, at each point x in the specimen. Two contributions will be considered. The first, $(\partial\rho/\partial t)_a$, is due to the ageing of mobile dislocations by solutes (or, in general, the stopping of dislocations by non-activable obstacles) for which we use the simple decay equation

$$\left(\frac{\partial\rho}{\partial t}\right)_a = -\frac{\rho}{\tau_a} \tag{8}$$

where τ_a is a characteristic time interval for ageing: the mobile dislocation density falls to $1/e$ of its initial value in τ_a.

The second source of variation of ρ with time at a given position, $(\partial\rho/\partial t)_s$, is related to the spreading of the mobile dislocations. Sändstrom and Lagneborg [18] have noted that this condition is crucial for Lüders band propagation and introduced a somewhat arbitrary broadening function to take into account the effect.

Simultaneously with the broadening of the dislocation distribution there may be an effect of stress concentration or amplification produced by the net incoming dislocations (due, for example, to dislocation pile-ups [19]). The inclusion of this effect in the constitutive equations could be achieved by defining an effective stress which differs from σ by a term that depends on $\partial\rho/\partial x$. In this approach, the strain rate would depend on $\partial\rho/\partial x$, and the rate equation would be of general form (Equation 3a), with a dependence of $\dot{\epsilon}$ on $\partial S/\partial x$. The stress concentration may produce fresh dislocations (by unpinning or by activation of sources) but we shall admit below that the dislocation velocity is unaffected.

Let us now introduce in the constitutive equations the effects of broadening and fresh dislocation generation. To do this, we first admit that the total length of mobile dislocation segments is conserved, in which case we may write:

$$\left(\frac{\partial\rho}{\partial t}\right)_s + \operatorname{div}(\rho \mathbf{v}) = 0 \tag{9}$$

which is the usual form of the conservation (or continuity) law for a fluid of density ρ and velocity **v**. To account for the generation of fresh dislocations we should introduce an extra term, $\dot{\rho}_g$, in Equation 9, but this would make difficult the integration of this equation. We prefer to intro-

duce a factor k in div(ρv), the value of which will be discussed below. In the one-dimensional model that we are considering, the contribution of dislocation spreading and generation to $\partial\rho/\partial t$ is therefore written as

$$\left(\frac{\partial\rho}{\partial t}\right)_{s,g} = -k v \frac{\partial\rho}{\partial x} \tag{10}$$

since we assume a constant velocity for the dislocations.

Combining Equations 8 and 10 we obtain for the total variation of ρ with time the following equation

$$\frac{\partial\rho}{\partial t} = -\frac{\rho}{\tau_a} - k v \frac{\partial\rho}{\partial x}. \tag{11}$$

Equations 7 and 11 are an example of the general form of the constitutive law indicated in Section 2 (Equations 3) for inhomogeneous deformation; the mobile dislocation density, ρ, is the structure variable.

3.2. Steady state Lüders band

Equation 11, with τ_a, k and v constants, admits a steady state solution, $\rho = \rho(x - v_L t)$, which represents a deformation state that travels at a constant velocity v_L in the direction $+x$. Writting

$$\xi = x - v_L t \tag{12}$$

Equation 11 is transformed into

$$(v_L - k v)\frac{d\rho}{d\xi} = \frac{\rho}{\tau_a} \tag{13}$$

which integrates to

$$\rho = \rho_0 \exp\left(-\frac{x - v_L t}{L}\right)$$
$$\text{where } L = (k v - v_L)\tau_a. \tag{14}$$

This deformation state does not represent a Lüders band, since, at fixed time, ρ (and therefore $\dot{\epsilon}$) varies monotonically with x. In a Lüders band there is a section x_L where $\dot{\epsilon}$ is larger than in the rest of the band; when the band moves with a velocity v_L it is $x_L = v_L t$, if the band is at the origin for $t = 0$. To meet this situation we have to use different solutions of the type (Equation 14), with different values of τ_a and k, ahead and behind the band. We assume however that in each region τ_a and k are constants. It is easily seen that the required solution is

$$\rho = \rho_0 \exp\left(-\frac{x - v_L t}{L_1}\right) \quad x \geqslant v_L t \tag{15a}$$

$$\rho = \rho_0 \exp\left(\frac{x - v_L t}{L_2}\right) \quad x \leqslant v_L t \tag{15b}$$

where the first equation applies ahead of the band and the second behind the band, and L_1 and $-L_2$ are the values of L in the two regions. Both L_1 and L_2 have to be positive for Solution 15 to be physically acceptable. Ahead of the band $k = k_1$ is larger than unity, but behind the band $k = k_2 < 1$ because $(\partial\rho/\partial x) < 0$ in this region (cf. Equation 10). When necessary, we shall assume that τ_a is the same everywhere (in fact, τ_a should be larger ahead of the band where the density of forest dislocations is smaller). Then

$$L_1 = (k_1 v - v_L)\tau_a$$
$$L_2 = (v_L - k_2 v)\tau_a. \tag{16}$$

Because $L_1, L_2 > 0$, the band velocity v_L must be in the interval $k_2 v, k_1 v$. We write

$$v_L = f v \tag{17}$$

where $k_2 < f < k_1$. This relation, with $f \simeq 1$, was first proposed by Hahn [20]. Other treatments [18] lead to f between 0.2 and 0.8.

It should be noted that the Solution 15, of the equations for the variation of ρ, represents a steady-state band because it was admitted that the band peak x_L moves with a velocity v_L. A moving band cannot be obtained from the simplified Equation 11 without this additional requirement. Even so, the band velocity is not determined by the model; any velocity between $k_2 v$ and $k_1 v$ is acceptable. A steady-state moving band could eventually be obtained as a solution of the general form of Equation 11 with τ_a and k functions of ρ and v function of ρ and $\partial\rho/\partial x$. The form of these functions is, however, poorly known.

3.3. Band parameters

We can now find various properties and parameters of the band described by Equations 7 and 15. The variation of $\dot{\epsilon}$ with x when the band is at the positon x_L is given by

$$\dot{\epsilon}(x) = \rho_0 b v \exp\left(-\frac{x - x_L}{k^* L_2}\right) \quad x \geqslant x_L \tag{18a}$$

$$\dot{\epsilon}(x) = \rho_0 b v \exp\left(\frac{x - x_L}{L_2}\right) \quad x \leqslant x_L \tag{18b}$$

where we introduced

$$k^* = \frac{L_1}{L_2}. \tag{19}$$

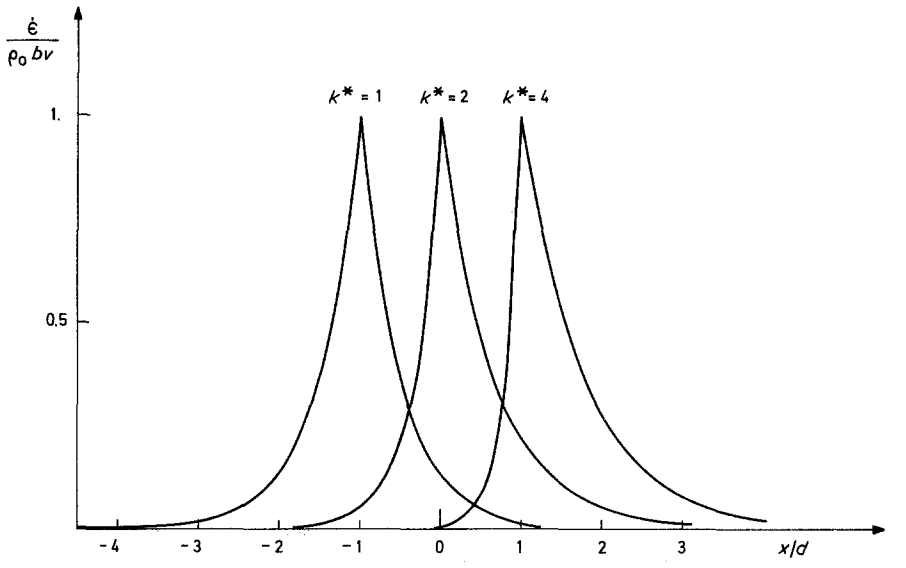

Figure 1 Strain rate profiles $\dot{\epsilon}(x)$ at a Lüders band for various values of the parameter k^*.

In general, the strain rate is larger ahead of the band than behind it, for the same distance to the band peak, implying $k^* > 1$. Fig. 1 shows plots of the strain rate profile for two values of $k^* > 1$. Also shown is the symmetric profile for $k^* = 1$. The reference length d in the abcissa is the band width (see below, Equation 25).

At a fixed position, $x = 0$ for example, the strain rate varies with time according to

$$\dot{\epsilon}(x=0;t) = \rho_0 b v \exp\left(\frac{v_L t}{k^* L_2}\right) \quad t \leqslant 0 \quad (20a)$$

$$\dot{\epsilon}(x=0;t) = \rho_0 b v \exp\left(-\frac{v_L t}{L_2}\right) \quad t \geqslant 0 \quad (20b)$$

Fig. 2 shows the variation of $\dot{\epsilon}$ with time, at a given section, for three values of k^*.

The accumulated strain at a fixed position ($x = 0$) is obtained from Equations 20 in the following way

$$\epsilon(x=0;t) = \int_{-\infty}^{t} \dot{\epsilon}\, dt = \rho_0 b v \frac{k^* L_2}{v_L} \times \exp\left(\frac{v_L t}{k^* L_2}\right) \quad t \leqslant 0 \quad (21a)$$

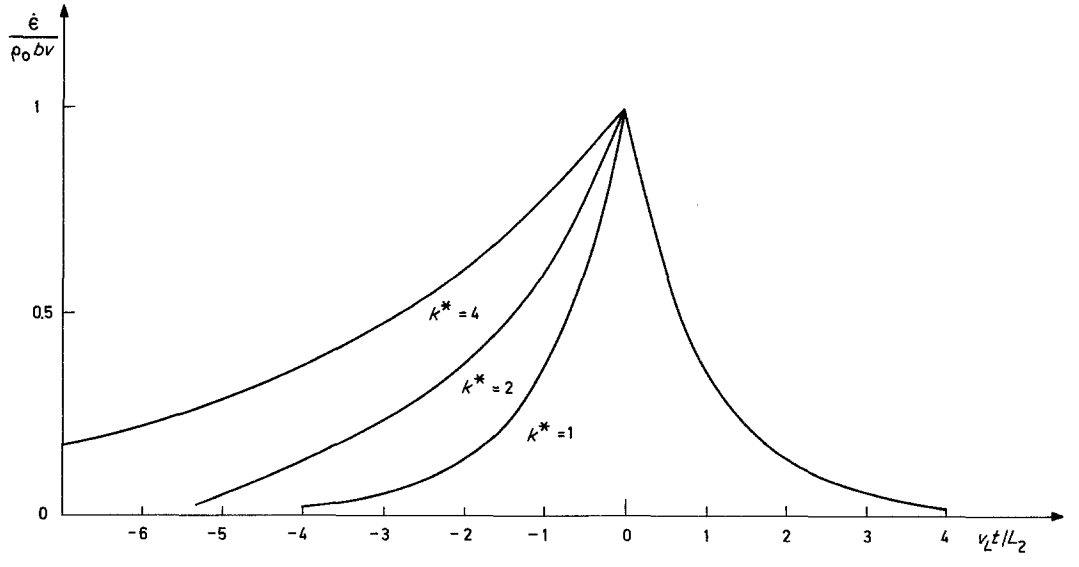

Figure 2 Strain rate at a fixed section as a function of time due to the passage of a Lüders band, for various values of the parameter k^*. The band peak crosses the section at $t = 0$.

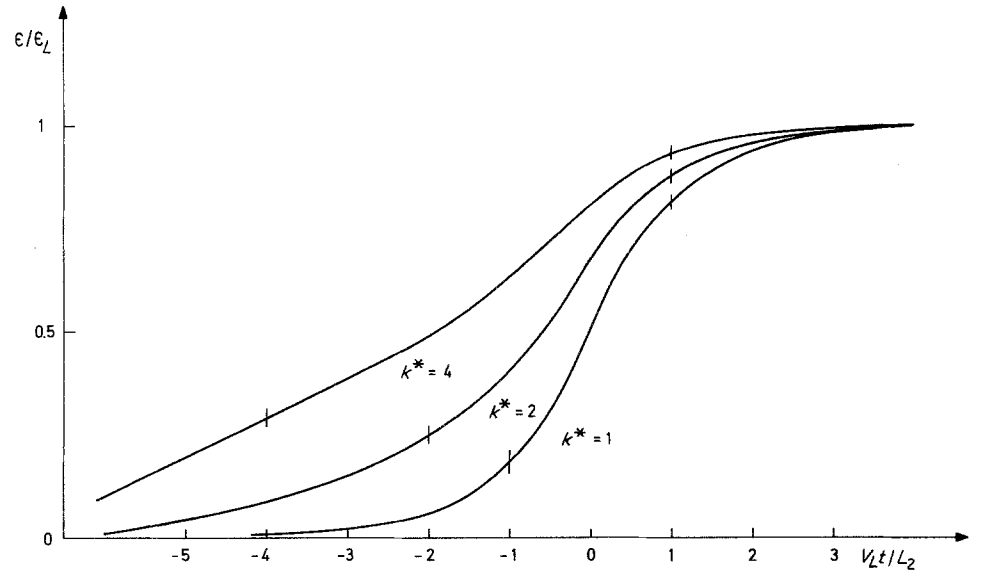

Figure 3 Strain at a fixed section as a function of time for various values of k^*. The vertical bars indicate the instants at which the strain rate is $1/e$ of its value at the peak and their separation is therefore proportional to the band width.

$$\epsilon(x=0;t) = \rho_0 b v \frac{k^* L_2}{v_L} + \int_0^t \dot{\epsilon}\, dt$$

$$= \rho_0 b v \frac{L_2}{v_L}\left[1 + k^* - \exp\left(-\frac{v_L t}{L_2}\right)\right] \quad t \geqslant 0 \tag{21b}$$

Fig. 3 shows plots of ϵ as a function of t obtained from Equations 21 for various values of k^*. These curves can be compared with those recorded by a transversal extensometer placed at a section of a specimen (see below).

The band strain, ϵ_L, is obtained by making $t = \infty$ in Equation 21b:

$$\epsilon_L = \rho_0 b v \frac{L_2}{v_L}(1 + k^*). \tag{22}$$

The rate of extension, $\dot{L}$, can be calculated from Equations 3 and 18 with, for example, $x_L = 0$

$$\dot{L} = \rho_0 b v \left[\int_0^\infty \exp\left(-\frac{x}{k^* L_2}\right) dx + \int_{-\infty}^0 \exp\left(\frac{x}{L_2}\right) dx\right] = \rho_0 b v L_2 (1 + k^*). \tag{23}$$

Combining Equations 22 and 23, we obtain the well-known relation

$$\dot{L} = \epsilon_L v_L. \tag{24}$$

The band width, d, can be defined (arbitrarily but consistently) as the separation between two sections, on opposite sides of the band peak, where $\dot{\epsilon}$ is a fraction $1/e$ of its value at the peak. From Equation 25 we find

$$d = L_1 + L_2 = L_2(1 + k^*). \tag{25}$$

We may also define an average band strain rate, $\dot{\epsilon}_L$, such that $\dot{L} = d\dot{\epsilon}_L$:

$$\dot{\epsilon}_L = \rho_0 b v. \tag{26}$$

The decay time τ_a depends on the velocity v of the dislocations, for a given structure. In the theories of strain ageing [21, 22] it is admitted that the dislocations are aged while waiting at obstacles, and a characteristic waiting time, t_w, is introduced. When serrated yielding occurs, this time is comparable to an ageing time, t_a, which depends on the kinetics of diffusion of the solute to the waiting dislocations [21, 22]. The ageing time t_a is distinct from τ_a, although it is probably proportional to τ_a.

In order to compare the predictions of the treatment presented with experimental results, it is convenient to introduce a characteristic length, λ, defined by

$$\lambda = v \tau_a \tag{27}$$

which is the average length travelled by the dislocations in time τ_a. The length λ can be defined even if immobilization is not due to the formation of solute atmospheres at the dislocations. It has the advantage over τ_a of being independent of v, for a given structure. Assuming that τ_a is the

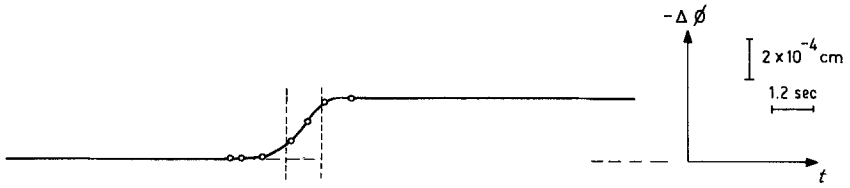

Figure 4 Experimental curve, $\Delta\phi(t)$, obtained with a tranversal extensometer and calculated points (see text). The bars indicate the time interval corresponding to the band width.

same ahead and behind the band peak, and using Equations 16, we may rewrite Equation 25 in terms of λ as

$$d = \Delta k \lambda \qquad (\Delta k = k_1 - k_2). \tag{28}$$

Using Equations 17 and 25, the Equations 22 and 23 for ϵ_L and $\dot{L}$ can be written in the form

$$\epsilon_L = \rho_0 \frac{b}{f} d \tag{29}$$

$$\dot{L} = \rho_0 b \frac{v_L d}{f}. \tag{30}$$

3.4. Comparison with experimental results

There are two types of Lüders band deformation in solids to which the previous treatment can be applied. The first type occurs in normal discontinuous yielding (e.g. in low carbon steels) where one band is formed near one grip and then travels under a fairly constant load (at constant $\dot{L}$) to the other end of the specimen. Homogeneous deformation occurs after this regime, which suggests that τ_a is larger or comparable to the time of transit of the band, so that a reasonable density of mobile dislocations is available after band propagation.

The second type is the one that leads to periodically serrated tensile curves (type A Portevin-Le Chatelier effect). Successive bands travel from one specimen end to the other, under a slightly increasing load at constant $\dot{L}$. This increase in load is due to a strain gradient that develops in the specimen [23, 24] as a result of a larger strain rate ahead of a moving band. It is possible, under special conditions [25], to eliminate the strain gradient, in which case successive bands propagate at increasing but constant stresses. The ageing time τ_a in this second type of band propagation is expected to be smaller than the time of transit of each band.

The accumulated strain derived from the model (Equations 21 and Fig. 3) can be directly compared with transversal extensometer curves. Such curves give the local diameter change, $\Delta\phi$, at a section as a function of time. Because the band strain is small, $\Delta\phi$ is proportional to ϵ. Fig. 4 shows an example of an extensometer curve (full curve) obtained with a commercial aluminium alloy (wire specimen dimensions: 6.8 cm; 0.10 cm^2; extension rate, 3.3×10^{-4} cm sec^{-1}) which upon suitable heat treatment [26] exhibits type A serrated curves. Similar extensometer curves have been obtained by van Westrum and Wijler [27]. The marked points in Fig. 4 were calculated from Equation 21 with $k^* = 1.5$ and $L_2/v_L = 0.48$ sec. Using the measured value $v_L = 0.23$ cm sec^{-1} (transit time approximately 30 sec) we find $L_2 = 0.093$ cm, $L_1 = 0.140$ cm and a band width of 2.4 mm. It is not possible to calculate τ_a because k_1 and k_2 are undetermined. The vertical dashed lines in Fig. 4 correspond to the time interval that a signal (the band) of width d takes to pass through a particular section and are centred at the instant of passage of the band peak. This stresses the arbitrariness in the definition of the band width and the associated difficulty of obtaining this parameter from transversal extensometer curves.

In materials exhibiting type A serrated curves, both the width and the velocity decrease for the successive bands, at constant $\dot{L}$ [24, 26–28]. Equation 28 predicts this effect on d, since the average length λ certainly decreases with accumulated strain. The decrease in v_L corresponds to a decrease in v, which occurs (in spite of the increasing stress in normal type A effect) because of strain hardening.

It is also experimentally found [27, 28] that both ϵ_L and d slightly increase with $\dot{L}$ (for the same accumulated strain). In one case [29], the width was found to be independent of $\dot{L}$. This indicates that λ may increase slightly with applied rate. The increase in λ and a similar effect on ρ_0 in Equation 29, may explain the increase of ϵ_L with $\dot{L}$.

Equation 29, with $f = 1$, has been derived by other methods [24, 31] and used [24, 29–31] to

calculate mobile dislocation densities at Lüders bands from measured values of ϵ_L (or v_L) and d.

Finally we refer briefly some results related to Lüders bands in discontinuous yielding. The band strain, ϵ_L, was found to be independent of applied extension rate [31] in agreement with Equation 28 and 29, and inversely proportional to the grain size [31, 32]. This suggests that λ is inversely proportional to the average grain diameter. Lloyd and Morris [31] identified these two quantities and used an equation similar to Equation 29 to calculate mobile dislocation densities. A strong dependence of v_L on $\dot{L}$ was reported in this work; this dependence is of course directly related to the strong effect of σ on dislocation velocity.

4. Discussion

The general form of a constitutive law for inhomogeneous flow contains extra variables in relation to the equations for homogeneous deformation. The extra variables are the gradients of the scalar structure variables; in general they appear both in the rate equation and in the equation for the time rate of change of the structure variables. This generalized form of the constitutive equations should be used in theoretical studies of plastic instability (necking) and is obviously the one appropriate to describe Lüders band deformation.

Hart [33] has analysed the inhomogeneous deformation of a tensile specimen by simply considering the static equilibrium of forces along the specimen (to obtain $\sigma(x)$) and assuming a constitutive equation of the type $\sigma(\dot{\epsilon}, \epsilon)$, which is of course questionable. If his results (Equation 3 in [33]) are applied to a strain rate insensitive material, it is found that $d\epsilon/dx = 0$, implying that in a specimen of such a material a strain gradient can never develop. There is, in fact, in Hart's treatment an incorrect identification of the strain increment as a function of position with the (time dependent) strain increment produced by deformation at each section. Hart's final equation for a steady state band (Equation 5 in [33]) is therefore incorrect.

The strain rate profile at a steadily moving Lüders band has been obtained in the present work by integration of simplified constitutive equations of the required form, in which the mobile dislocation density plays the role of the structure variable. The constitutive equations were written based on a balance of the dislocation density at each point, which takes into account the spreading, ageing and generation of dislocations. With the simplifications introduced, it is not possible to predict the formation of a Lüders band and its transition to a steady or quasi-steady propagation regime. It was also not attempted to explain the load drops associated with the nucleation of bands. The inclusion of these effects has been attempted by Sändstrom and Lagneborg [18].

The model was developed for a band propagating under a constant stress and the steady state solution obtained shows a strain rate profile extending to infinity. In spite of the simplifications introduced, the calculated strain profile as a function of time at a given location compares very well with that recorded by a transversal extensometer placed on a specimen exhibiting the type A Portevin–Le Chatelier effect, when the band propagates under a slightly increasing load in a strain gradient and, therefore, is only approximately a steady state band.

The properties of the band parameters obtained from the model also compare very well with those found experimentally. Finally, it is remarkable that the model leads to an often quoted relation between the Lüders strain, the band width and the mobile dislocation density (Equation 29).

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